PRACTICE PROBLEMS FOR MIDTERM 1
MATH 430, SPRING 2015

Problem 1. *Determine if the following are tautologies:*
   (a) $(R \to (S \lor Q)) \lor (R \lor (S \to Q))$
   (b) $(R \leftrightarrow P) \lor (P \to \neg R)$

Problem 2. *Prove or refute the following:*
   (a)If $\Sigma \models (\alpha \land \beta)$, *then* $\Sigma \models \alpha$ *and* $\Sigma \models \beta$
   (b)If $\Sigma \models (\alpha \lor \beta)$, *then* $\Sigma \models \alpha$ *or* $\Sigma \models \beta$

Problem 3. *Let S be the set of all sentence symbols, and let $\bar{S}$ be the set of all (sentential) formulas built up from S. Fix a truth assignment $\nu : S \to \{T, F\}$. Without using the recursion theorem, show that there is at most one extension $\bar{\nu} : \bar{S} \to \{T, F\}$ satisfying the truth table conditions.*

Problem 4. *Show that $\{\land, \leftrightarrow, +\}$ is complete, but $\{\land, +\}$ is not complete. Here $\alpha + \beta$ means $(\alpha \lor \beta) \land \neg(\alpha \land \beta)$ i.e. either α or β is true, but not both.*

Problem 5. *Show that $\{\bot, \to\}$ is complete, but $\{\land, \to\}$ is not complete.*

Problem 6. *Show that $\{\to, +\}$ is complete, but $\{\leftrightarrow, +\}$ is not complete.*

Problem 7. *Recall that the corollary to the Compactness theorem states that if $\Sigma \models \tau$, then there is some finite $\Delta \subset \Sigma$ such that $\Delta \models \tau$. Show that the Compactness theorem is equivalent to this corollary. (Prove both directions.)*

Problem 8. *Suppose Σ is satisfiable and complete (i.e. for every formula ϕ, either $\phi \in \Sigma$ or $\neg\phi \in \Sigma$). Define a truth assignment v by $v(A) = T$ iff $A \in \Sigma$. Show that for each $\phi \in \Sigma$, $\bar{v}(\phi) = T$ iff $\phi \in \Sigma$.*

Problem 9. *Write the following in the first order language of set theory, $\mathcal{L} = \{\in\}$:*
   (a) z is the emptyset
   (b) x is a subset of y.
   (c) The power set of x is equal to the power set of y. (The power set of x, denoted $\mathcal{P}(x)$, is the set of all subsets of x.)
   Which of the above formulas contains a free variable, and which one is it?